Rhombus, Rectangle and squares (pp 347-350)

- I can define a rectangle, rhombus and square.
- I can state the properties of rectangles, rhombi & squares.
- I can use rectangles, rhombi & squares to solve problems and complete proofs.

COROLLARIES ABOUT SPECIAL QUADRILATERALS

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

SQUARE COROLLARY

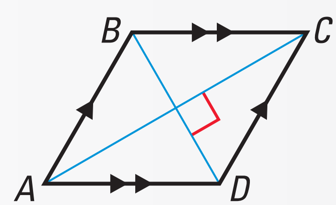
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

THEOREMS

THEOREM 6.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

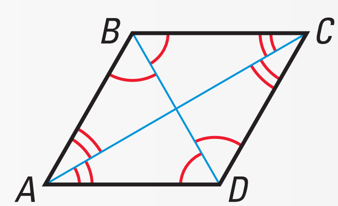
ABCD is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.



THEOREM 6.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

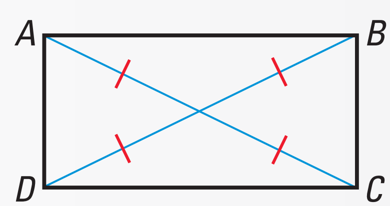
ABCD is a rhombus if and only if
$\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$ and
$\overline{BD}$ bisects $\angle ADC$ and $\angle CBA$.



THEOREM 6.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

ABCD is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.



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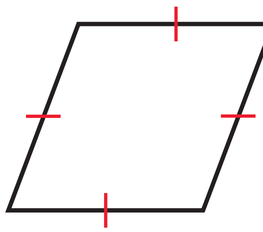
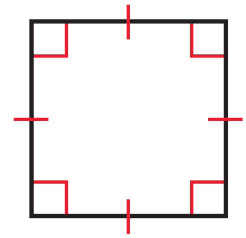


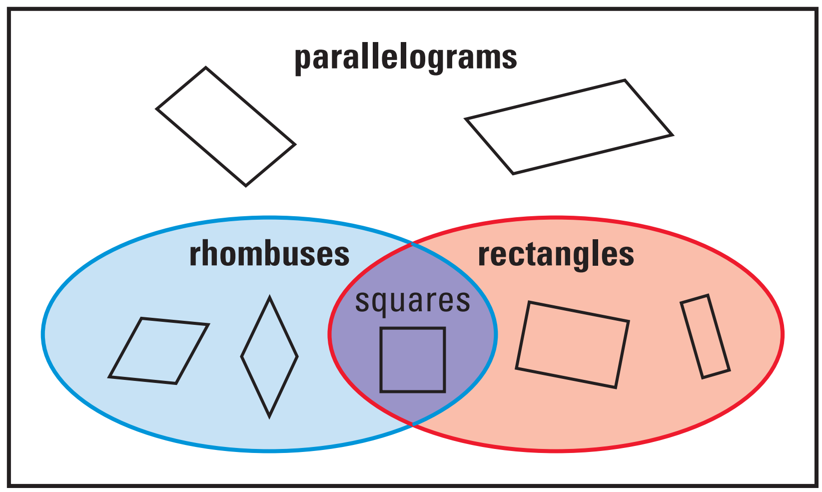
A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

The *Venn diagram* at the right shows the relationships among parallelograms, rhombuses, rectangles, and squares. Each shape has the properties of every group that it belongs to. For instance, a square is a rectangle, a rhombus, and a parallelogram, so it has all of the properties of each of those shapes.



A rectangle is defined as a *parallelogram* with four right angles. However, *any equiangular quadrilateral* is a rectangle because any quadrilateral with four right angles is a rectangle.

To prove an if and only if statement, you must prove the conditional and its converse.

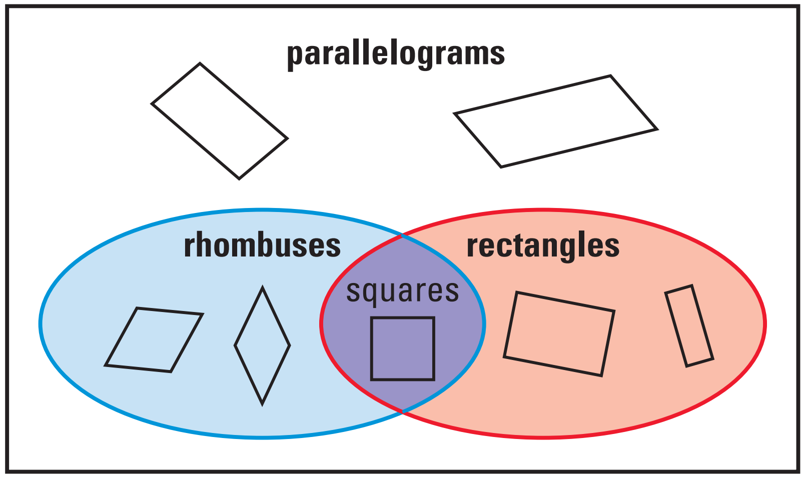
Rhombus, Rectangle and squares (pp 347-350)

EXAMPLE 1 Describing a Special Parallelogram

Decide whether the statement is *always*, *sometimes*, or *never* true.

a. A rhombus is a rectangle.

b. A parallelogram is a rectangle.



SOLUTION

a. The statement is *sometimes* true. In the Venn diagram, the regions for rhombuses and rectangles overlap. If the rhombus is a square, it is a rectangle.

b. The statement is *sometimes* true. Some parallelograms are rectangles. In the Venn diagram, you can see that some of the shapes in the parallelogram box are in the region for rectangles, but many aren't.

Are the following always, sometimes or never true?

2. _____ A rectangle is a square.

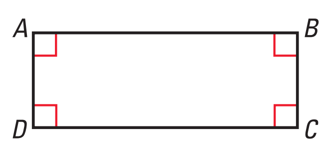
3. _____ A square is a rhombus.

4. _____ A rectangle is a parallelogram.

Logical Reasoning

EXAMPLE 2 Using Properties of Special Parallelograms

ABCD is a rectangle. What else do you know about *ABCD*?



SOLUTION

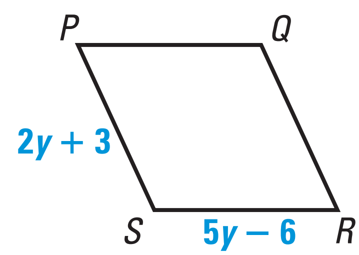
Because *ABCD* is a rectangle, it has four right angles by the definition. The definition also states that rectangles are parallelograms, so *ABCD* has all the properties of a parallelogram:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent and consecutive angles are supplementary.
- Diagonals bisect each other.

5. QRST is a square, what else do you know about QRST?

EXAMPLE 3 *Using Properties of a Rhombus*

In the diagram at the right, *PQRS* is a rhombus. What is the value of *y*?



SOLUTION

All four sides of a rhombus are congruent, so $RS = PS$.

$5y - 6 = 2y + 3$ **Equate lengths of congruent sides.**

$5y = 2y + 9$ **Add 6 to each side.**

$3y = 9$ **Subtract 2*y* from each side.**

$y = 3$ **Divide each side by 3.**

6. EFGH is a rectangle. K is the midpoint of $\overline{FH}$. If EG = 8z - 16, what is EK, GK?

7. ABCD is a rectangle, and $m\measuredangle B = 8x + 26$. What is the value of x.

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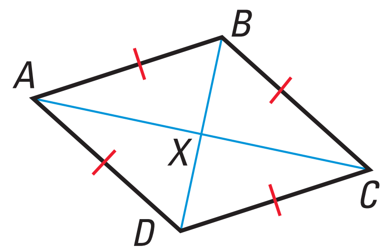
8. What properties does each special parallelogram that makes it different from a parallelogram?

Write a paragraph proof of the converse above.



GIVEN ▶ *ABCD* is a rhombus.

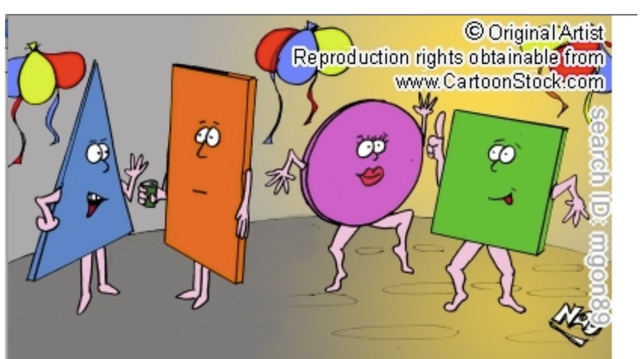
PROVE ▶ $\overline{AC} \perp \overline{BD}$

EXAMPLE 4

SOLUTION

Paragraph Proof *ABCD* is a rhombus, so $\overline{AB} \cong \overline{CB}$. Because *ABCD* is a parallelogram, its diagonals bisect each other so $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{BX}$. Use the SSS Congruence Postulate to prove $\triangle AXB \cong \triangle CXB$, so $\angle AXB \cong \angle CXB$. Then, because $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\overline{AC} \perp \overline{BD}$.

9. In $\square ABCD$, the diagonals meet at point E, and AE = BE = 6. Is ABCD a rectangle? Explain your solution.



"Hey, check out the square dancing with Brenda...!"

EXAMPLE 5 Coordinate Proof of Theorem 6.11

Proof

In Example 4, a paragraph proof was given for part of Theorem 6.11. Write a coordinate proof of the original conditional statement.

GIVEN ▶ $ABCD$ is a parallelogram, $\overline{AC} \perp \overline{BD}$.

PROVE ▶ $ABCD$ is a rhombus.

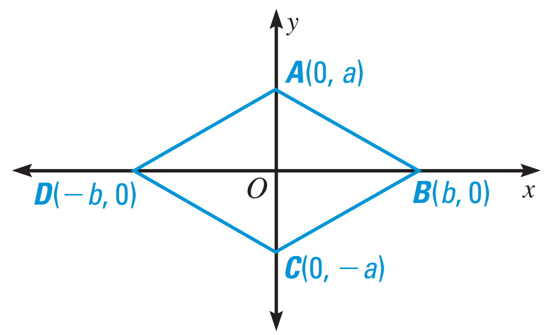
SOLUTION

Assign coordinates Because $\overline{AC} \perp \overline{BD}$, place $ABCD$ in the coordinate plane so $\overline{AC}$ and $\overline{BD}$ lie on the axes and their intersection is at the origin.

Let $(0, a)$ be the coordinates of A, and let $(b, 0)$ be the coordinates of B.

Because $ABCD$ is a parallelogram, the diagonals bisect each other and $OA = OC$. So, the coordinates of C are $(0, -a)$.

Similarly, the coordinates of D are $(-b, 0)$.



Find the lengths of the sides of $ABCD$. Use the Distance Formula.

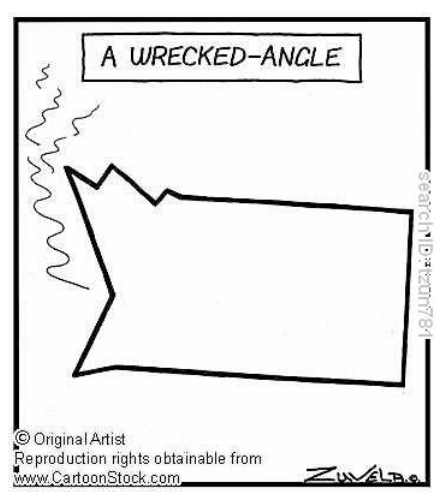
$$AB = \sqrt{(b-0)^2 + (0-a)^2} = \sqrt{b^2 + a^2}$$

$$BC = \sqrt{(0-b)^2 + (-a-0)^2} = \sqrt{b^2 + a^2}$$

$$CD = \sqrt{(-b-0)^2 + [0-(-a)]^2} = \sqrt{b^2 + a^2}$$

$$DA = \sqrt{[0-(-b)]^2 + (a-0)^2} = \sqrt{b^2 + a^2}$$

▶ All of the side lengths are equal, so $ABCD$ is a rhombus.



9. Write a coordinate Proof.

Given: ABCD is a ▱.
$\overline{AC} \perp \overline{BD}$.

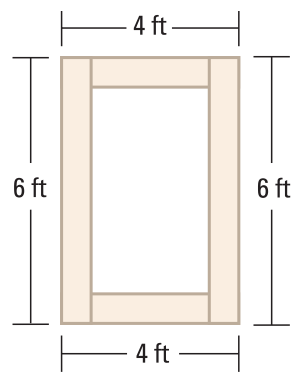
Proof: ABCD is rhombus.

FOCUS ON APPLICATIONS

CARPENTRY If a screen door is not rectangular, you can use a piece of hardware called a *turnbuckle* to shorten the longer diagonal until the door is rectangular.

EXAMPLE 6 ***Checking a Rectangle***

CARPENTRY You are building a rectangular frame for a theater set.



a. First, you nail four pieces of wood together, as shown at the right. What is the shape of the frame?

b. To make sure the frame is a rectangle, you measure the diagonals. One is 7 feet 4 inches and the other is 7 feet 2 inches. Is the frame a rectangle? Explain.

SOLUTION

a. Opposite sides are congruent, so the frame is a parallelogram.

b. The parallelogram is not a rectangle. If it were a rectangle, the diagonals would be congruent.

10. You cut out a parallelogram-shaped quilt piece and measure the diagonals appear to be congruent. What is the shape?

11. An angle formed by the diagonals of the quilt piece measures 90° . Is the shape a square?

12. In $\square RSTV$, the diagonals form a pair of congruent angles at each vertex. What kind of figure is *RSTU*?

13. What is true about the diagonals of a rectangle and a square, but not of those of every rhombus?

14. What is another name for an *equilateral quadrilateral*?

Decide whether the statement is *sometimes*, *always*, or *never* true.

15. _____ A rectangle is a parallelogram.

16. _____ A rectangle is a rhombus.

17. _____ A parallelogram is a rhombus.

18. _____ A square is a rectangle.